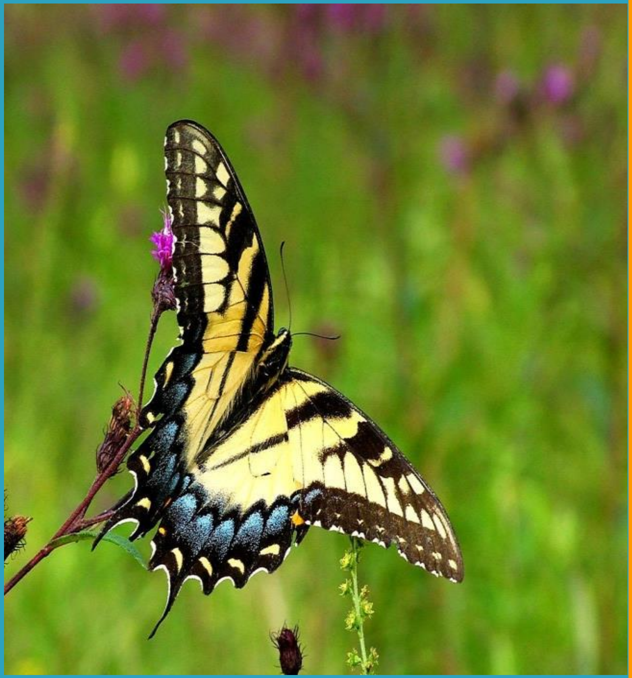
Wave Optics-I



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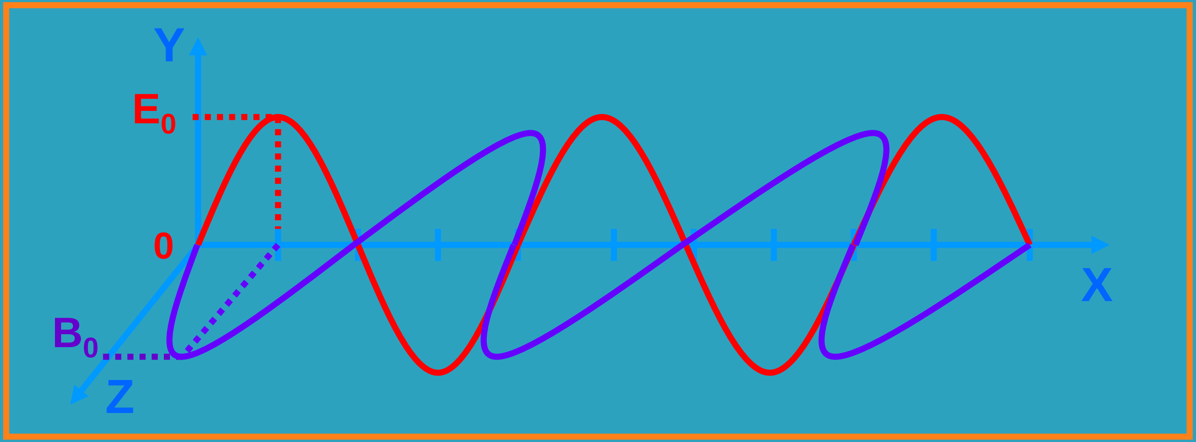
1. **Electromagnetic Wave**
2. **Wave front**
3. **Huygens' Principle**
4. **Reflection of Light based on Huygens' Principle**
5. **Refraction of Light based on Huygens' Principle**
6. **Behavior of Wavefront in a Mirror and Lens**
7. **Coherent Sources**
8. **Interference**

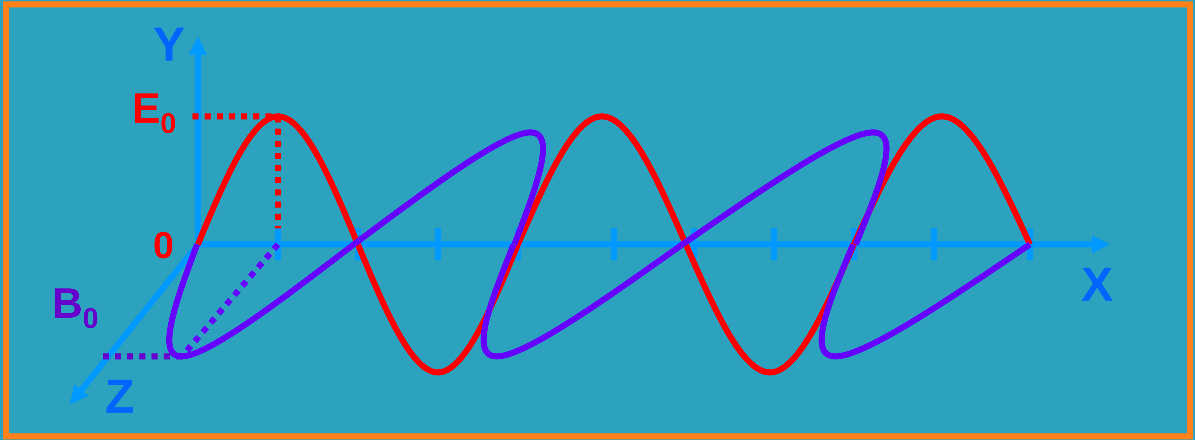
Electromagnetic Wave



1. Variations in both electric and magnetic fields occur simultaneously. Therefore, they attain their maxima and minima at the same place and at the same time.

2. The direction of electric and magnetic fields are mutually perpendicular to each other and as well as to the direction of propagation of wave.

Electromagnetic Wave



3. The speed of electromagnetic wave depends entirely on the electric and magnetic properties of the medium, in which the wave travels and not on the amplitudes of their variations.

Wave is propagating along X – axis with speed

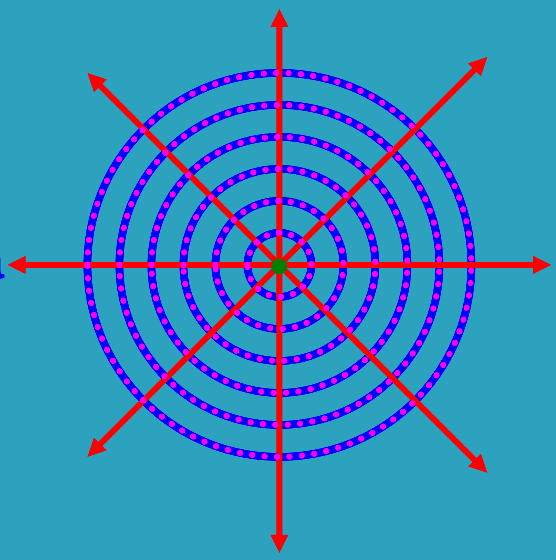
$$c = 1 / \sqrt{\mu_0 \varepsilon_0}$$

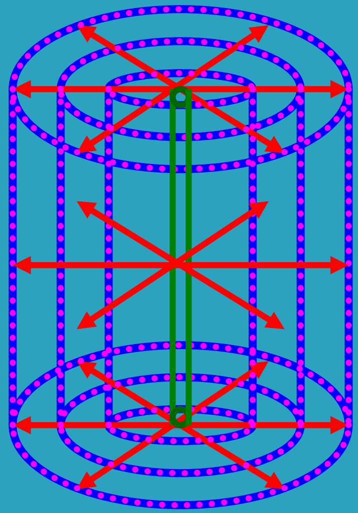
For discussion of optical property of EM wave, more significance is given to Electric Field, E. Therefore, Electric Field is called '**light vector**'.

Wavefront

A wavefront is the locus of points (wavelets) having the same phase of oscillations.

Spherical Wavefront from a point source





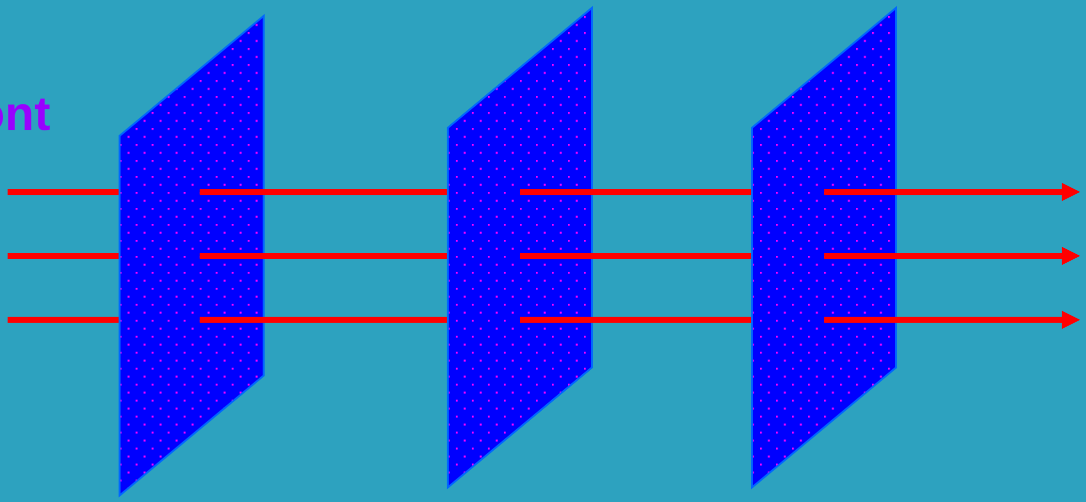
Cylindrical Wavefront from a linear source

A line perpendicular to a wavefront is called a 'ray'.

Wavelet

A wavelet is the point of disturbance due to propagation of light.

Plane Wavefront

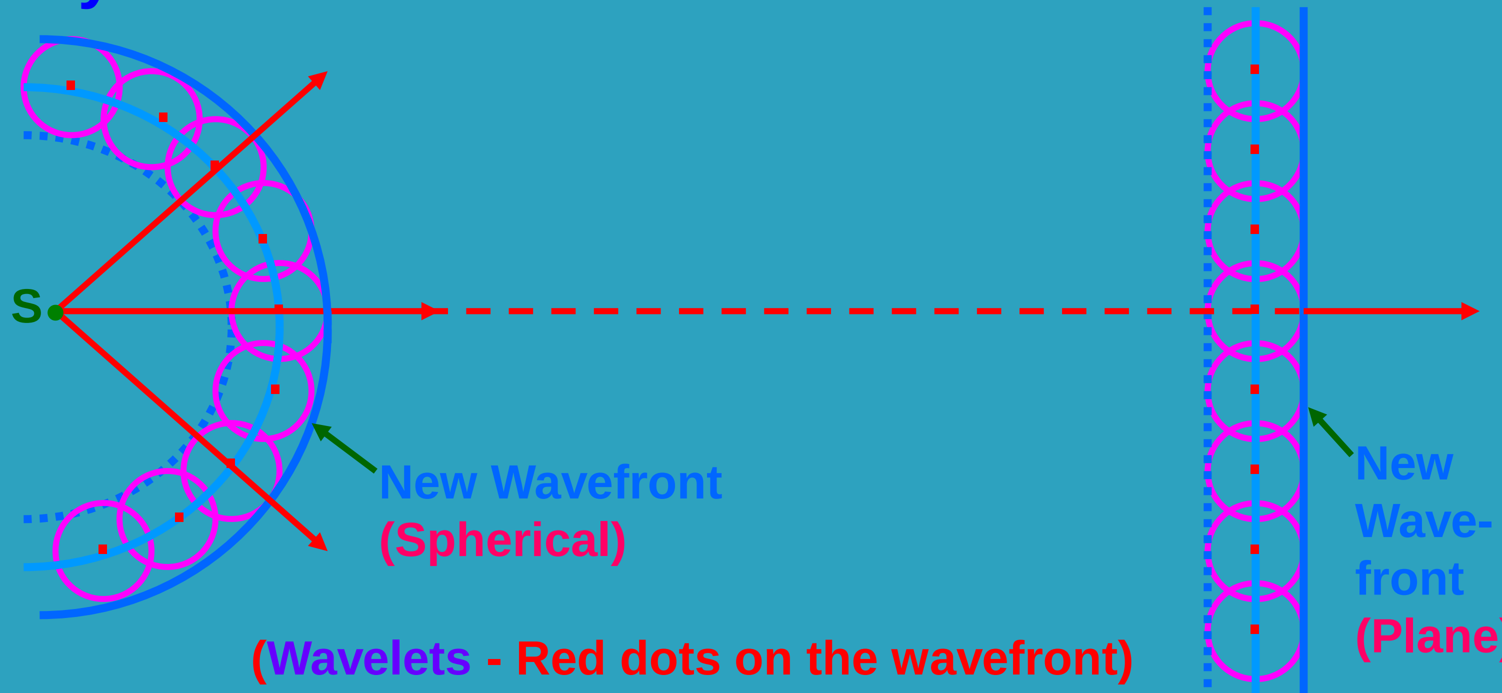


Pink Dots – Wavelets

Blue Envelope– Wavefront

Red Line – Ray

Huygens' Construction or Huygens' Principle of Secondary Wavelets:



1. Each point on a wavefront acts as a fresh source of disturbance of light.

2. The new wavefront at any time later is obtained by taking the forward envelope of all the secondary wavelets at that time.

Huygens' Construction or Huygens' Principle of Secondary Wavelets:

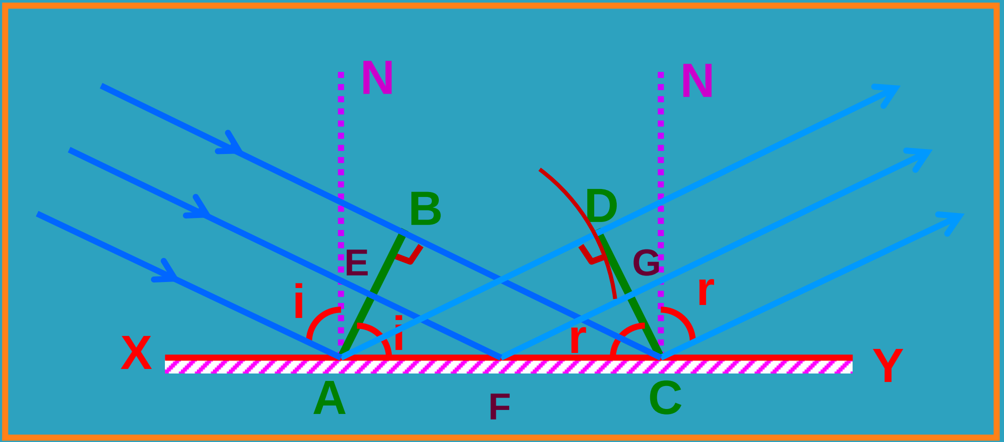
(Wavelets - Red dots on the wavefront)

Note: Backward wavefront is rejected. Why?

Amplitude of secondary wavelet is proportional to ½ (1+cosθ). Obviously, for the backward wavelet θ = 180° and (1+cosθ) is 0.

Laws of Reflection at a Plane Surface (On Huygens' Principle):

If c be the speed of light, t be the time taken by light to go from B to C or A to D or E to G through F, then



$$t = \frac{EF}{c} + \frac{FG}{c}$$

AB – Incident wavefront

CD – Reflected wavefront

XY – Reflecting surface

$$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$$

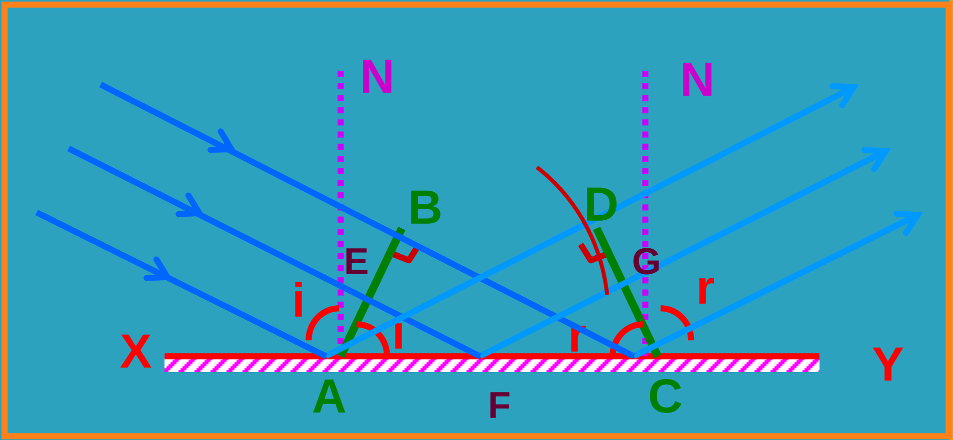
$$t = \frac{AC \sin r + AF (\sin i - \sin r)}{c}$$

Laws of Reflection at a Plane Surface (On Huygens' Principle):

AB – Incident wavefront

CD – Reflected wavefront

XY – Reflecting surface



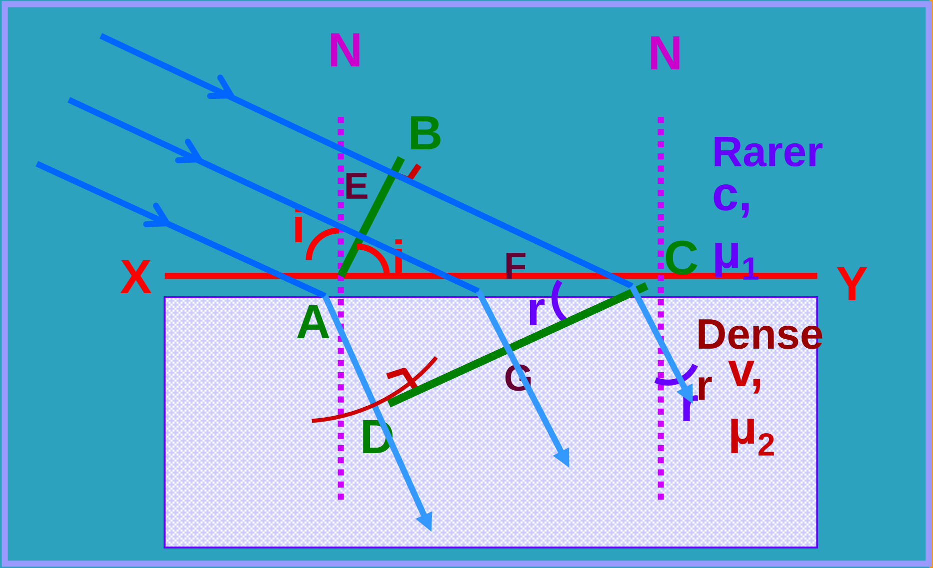
For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront. So, t should not depend upon AF. This is possible only if sin i – sin r = 0.

i.e. sin i = sin r or i = r

Laws of Refraction at a Plane Surface

(On Huygens' Principle):

If c be the speed of light, t be the time taken by light to go from B to C or A to D or E to G through F, then



$$t = \frac{EF}{c} + \frac{FG}{v}$$

$$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{v}$$

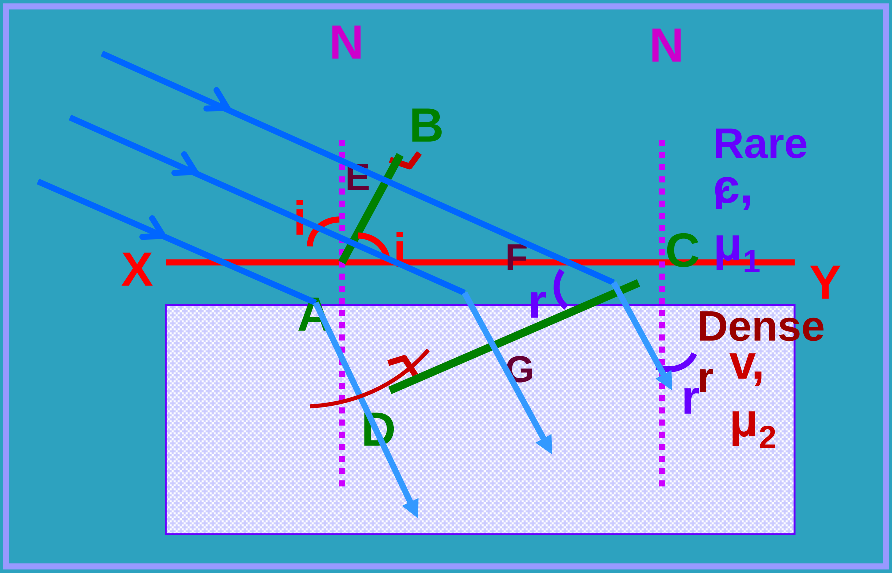
$$t = \frac{AC \sin r}{v} + AF\left(\frac{\sin i}{c} - \frac{\sin r}{v}\right)$$

AB – Incident wavefront
CD – Refracted wavefront
XY – Refracting surface

Laws of Refraction at a Plane Surface

(On Huygens' Principle):

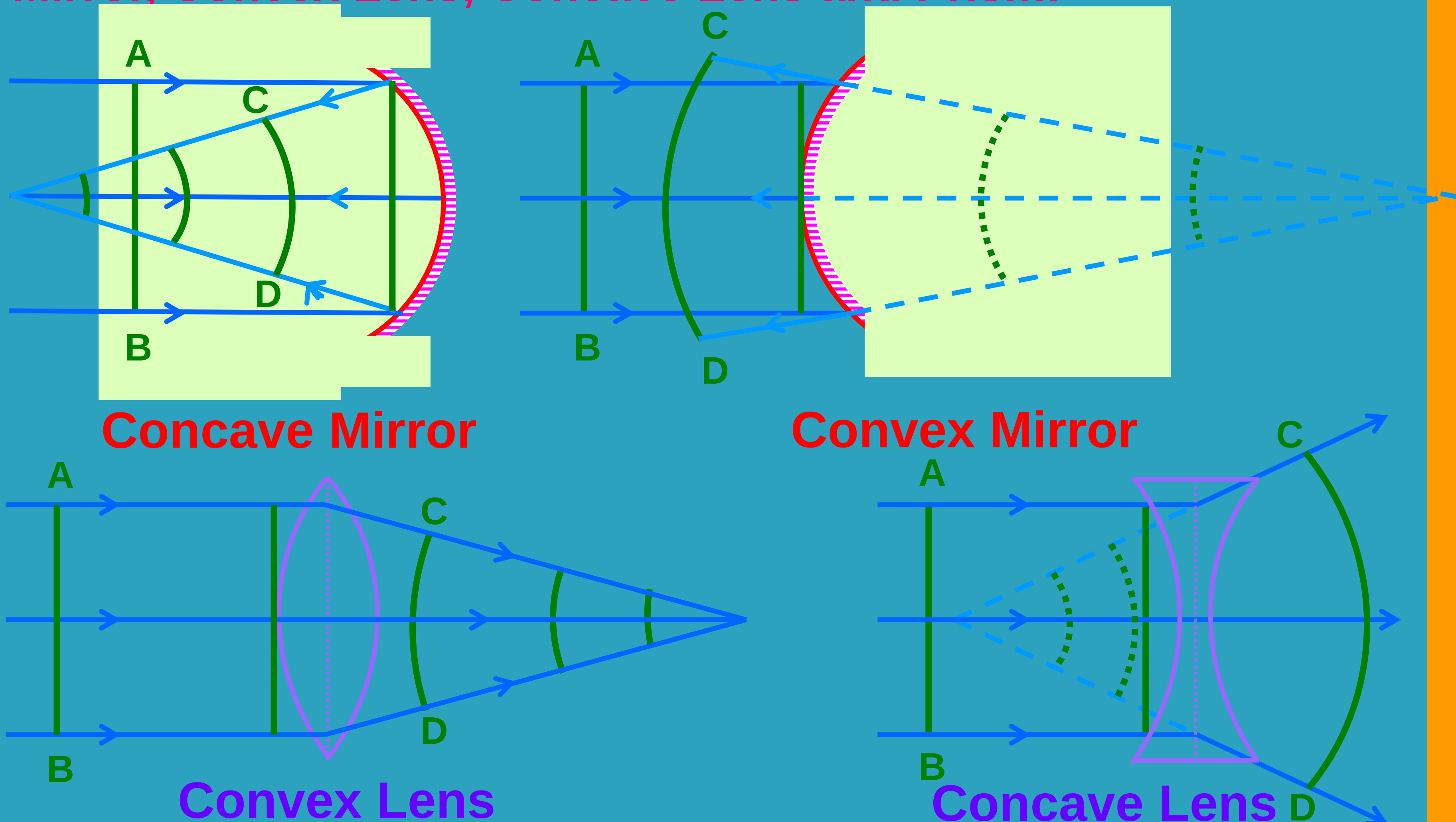
For rays of light from different parts on the incident wavefront, the values of AF are different.



But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront. So, t should not depend upon AF. This is possible only if

$$\frac{\sin i}{c} - \frac{\sin r}{v} = 0 \quad \text{or} \quad \frac{\sin i}{c} = \frac{\sin r}{v} \quad \text{or} \quad \frac{\sin i}{\sin r} = \frac{c}{v} = \mu$$

Behaviour of a Plane Wavefront in a Concave Mirror, Convex Mirror, Convex Lens, Concave Lens and Prism:



AB – Incident wavefront ,CD – Reflected /Refracted wavefront

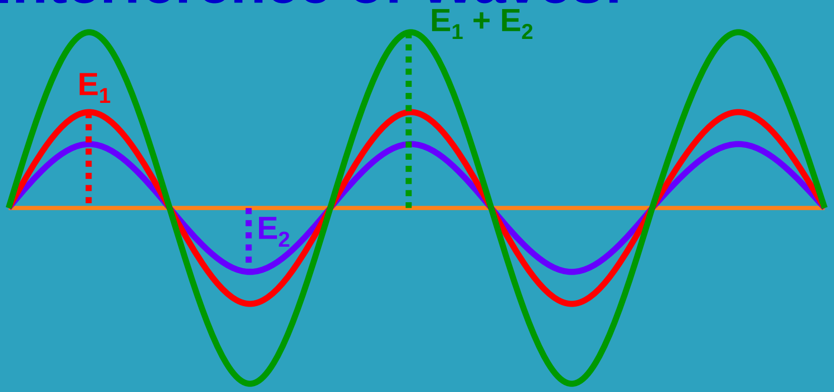
Coherent Sources

Coherent Sources of light are those sources of light which emit light waves of same wavelength, same frequency and in same phase or having constant phase difference.

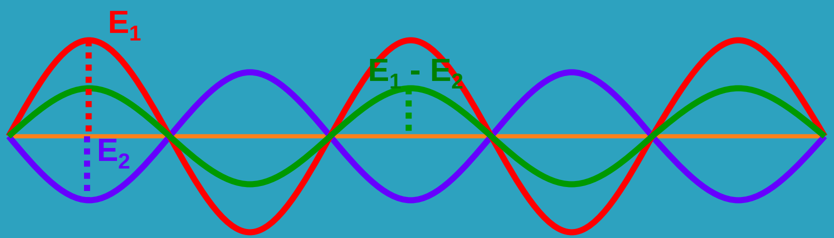
Coherent sources can be produced by two methods:

1. **By division of wavefront (Young's Double Slit Experiment, Fresnel's Biprism and Lloyd's Mirror)**
2. **By division of amplitude (Partial reflection or refraction)**

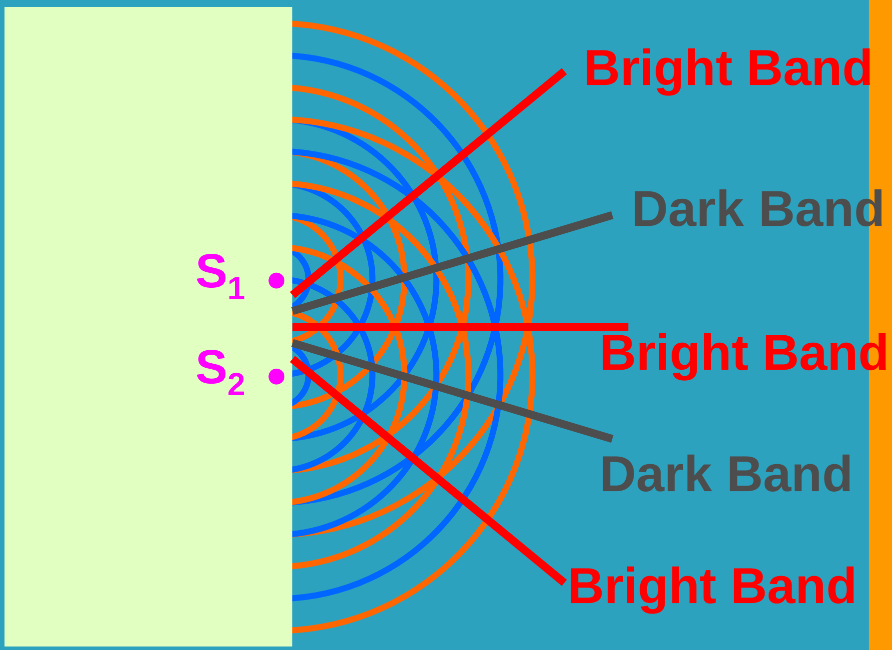
Interference of Waves:



Constructive Interference $E = E_1 + E_2$



Destructive Interference $E = E_1 - E_2$



Crest
Trough
Bright Band
Dark Band

1st Wave (E_1)
2nd Wave (E_2)
Resultant Wave
Reference Line

The phenomenon of one wave interfering with another and the resulting redistribution of energy in the space around the two sources of disturbance is called **interference of waves**.

Theory of Interference of Waves

The waves are with same speed, wavelength, frequency, time period, nearly equal amplitudes, travelling in the same direction with constant phase difference of Φ.

$$E_1 = a \sin \omega t$$

$$E_2 = b \sin (\omega t + \Phi)$$

ω is the angular frequency of the waves, a, b are the amplitudes and E_1, E_2 are the instantaneous values of Electric displacement.

Theory of Interference of Waves

$E = E_1 + E_2$

$E = a \sin \omega t + b \sin (\omega t + \Phi)$

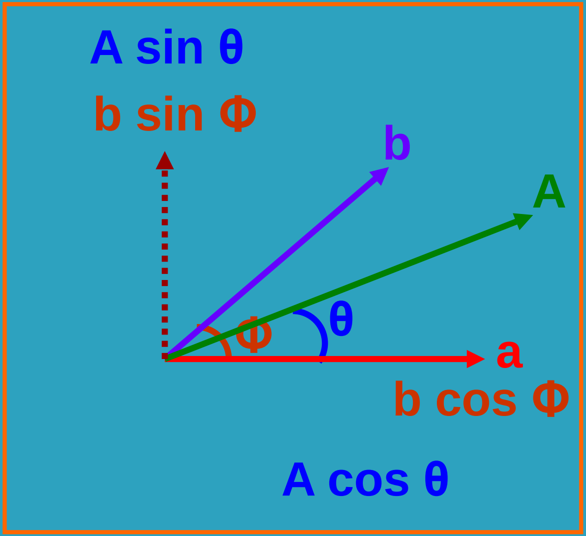
$E = (a + b \cos \Phi) \sin \omega t + b \sin \Phi \cos \omega t$

Putting $a + b \cos \Phi = A \cos \theta$

$b \sin \Phi = A \sin \theta$

We get $E = A \sin (\omega t + \theta)$

(where E is the resultant displacement, A is the resultant amplitude and θ is the resultant phase difference)



$$\boxed{A = \sqrt{(a^2 + b^2 + 2ab \cos \Phi)}}$$

$$\boxed{\tan \theta = \frac{b \sin \Phi}{a + b \cos \Phi}}$$

$$\boxed{A = \sqrt{(a^2 + b^2 + 2ab \cos \Phi)}}$$

Intensity I is proportional to the square of the amplitude of the wave.

So, $I \propto A^2$ i.e. $I \propto (a^2 + b^2 + 2ab \cos \Phi)$

Condition for Constructive Interference of Waves

For constructive interference, I should be maximum which is possible only if $\cos \Phi = +1$.

$\boxed{\text{i.e. } \Phi = 2n\pi}$ **where n = 0, 1, 2, 3, …….**

Corresponding path difference is $\Delta = (\lambda / 2\pi) \times 2n\pi$

$\boxed{\Delta = n\lambda}$

$\boxed{I_{max} \propto (a + b)^2}$

Condition for Destructive Interference of Waves:

For destructive interference, I should be minimum which is possible only if $\cos \Phi = -1$.

$\boxed{\text{i.e. } \Phi = (2n + 1)\pi}$

where n = 0, 1, 2, 3, …….

Corresponding path difference is $\Delta = (\lambda / 2\pi) \times (2n + 1)\pi$

$\boxed{\Delta = (2n + 1)\lambda / 2}$

$\boxed{I_{min} \propto (a - b)^2}$

Comparison of intensities of maxima and minima:

$$\boxed{I_{max} \;\alpha\; (a + b)^2}$$

$$\boxed{I_{min} \;\alpha\; (a - b)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(a + b)^2}{(a - b)^2} = \frac{(a/b + 1)^2}{(a/b - 1)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(r + 1)^2}{(r - 1)^2}$$

where $r = a / b$ (ratio of the amplitudes)

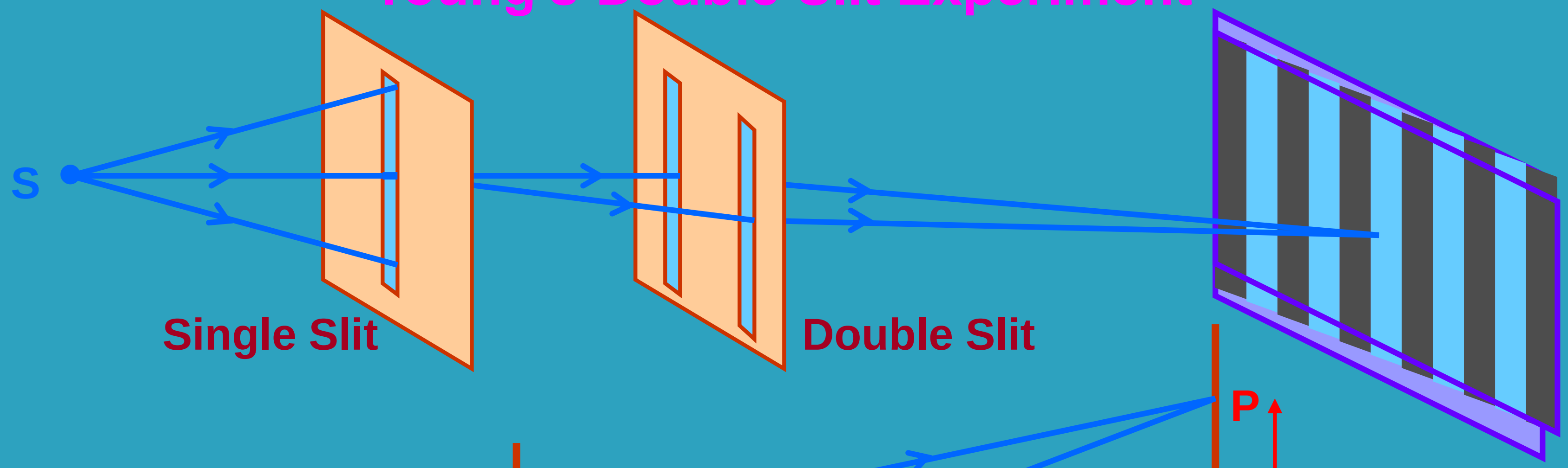
Relation between Intensity (I), Amplitude (a) of the wave and Width (w) of the slit:

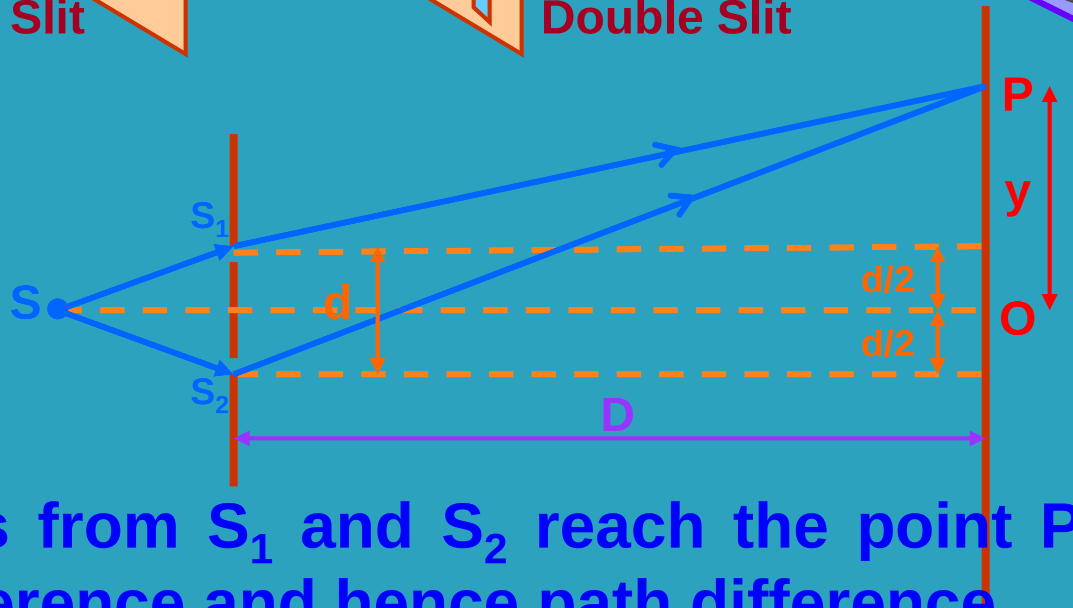
$$I \;\alpha\; a^2$$

$$a \;\alpha\; \sqrt{w}$$

$$\boxed{\frac{I_1}{I_2} = \frac{(a_1)^2}{(a_2)^2} = \frac{w_1}{w_2}}$$

Young's Double Slit Experiment





The waves from S_1 and S_2 reach the point P with some phase difference and hence path difference

$$\Delta = S_2P - S_1P$$

$$S_2P^2 - S_1P^2 = [D^2 + \{y + (d/2)\}^2] - [D^2 + \{y - (d/2)\}^2]$$

$$(S_2P - S_1P)\,(S_2P + S_1P) = 2\,yd \qquad \Delta\,(2D) = 2\,yd$$

$$\boxed{\Delta = yd / D}$$

Positions of Bright Fringes:

For a bright fringe at P,

$\Delta = yd / D = n\lambda$

where n = 0, 1, 2, 3,

$y = n D \lambda / d$

For n = 0, $y_0 = 0$

For n = 1, $y_1 = D \lambda / d$

For n = 2, $y_2 = 2 D \lambda / d$

…………

For n = n, $y_n = n D \lambda / d$

Positions of Dark Fringes:

For a dark fringe at P,

$\Delta = yd / D = (2n+1)\lambda/2$

where n = 0, 1, 2, 3, …

$y = (2n+1) D \lambda / 2d$

For n = 0, $y_0' = D \lambda / 2d$

For n = 1, $y_1' = 3D \lambda / 2d$

For n = 2, $y_2' = 5D \lambda / 2d$

…………

For n = n, $y_n' = (2n+1)D \lambda / 2d$

Expression for Dark Fringe Width: $\beta_D = y_n - y_{n-1}$

$= n\, D\, \lambda / d - (n - 1)\, D\, \lambda / d$

$= D\, \lambda / d$

Expression for Bright Fringe Width:

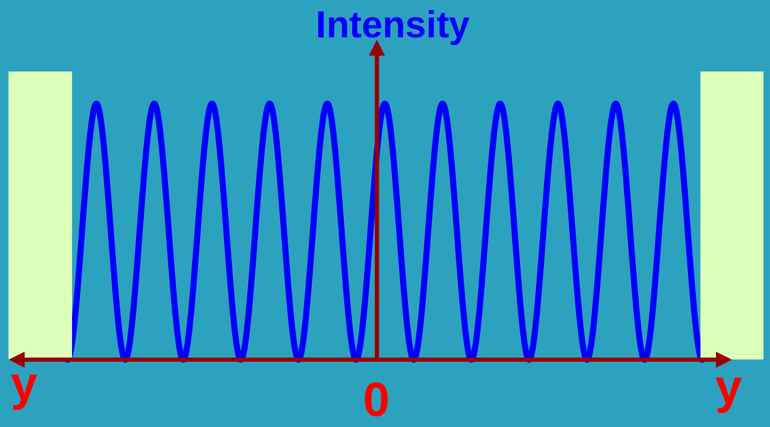
$\beta_B = y_n' - y_{n-1}'$

$= (2n+1)\, D\, \lambda / 2d - \{2(n-1)+1\}\, D\, \lambda / 2d$

$= D\, \lambda / d$

The expressions for fringe width show that the fringes are equally spaced on the screen.

Distribution of Intensity



Suppose the two interfering waves have same amplitude say 'a', then

I_{max} α $(a+a)^2$ i.e. I_{max} α $4a^2$

All the bright fringes have this same intensity.

$I_{min} = 0$

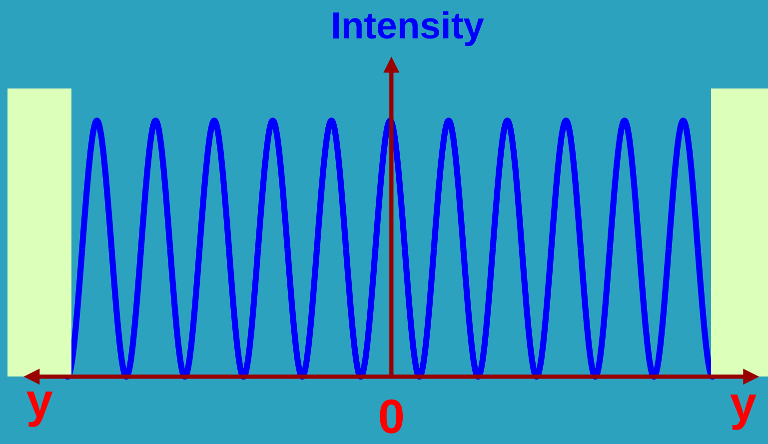
All the dark fringes have zero intensity.

Conditions for sustained interference

1. The two sources producing interference must be coherent.

2. The two interfering wave trains must have the same plane of polarisation.

Distribution of Intensity



Conditions for sustained interference:

3. The two sources must be very close to each other and the pattern must be observed at a larger distance to have sufficient width of the fringe. ($D \lambda / d$)

4. The sources must be monochromatic. Otherwise, the fringes of different colours will overlap.

5. The two waves must be having same amplitude for better contrast between bright and dark fringes.